

TARGET MATHEMATICS AGYAT GUPTA (M.Sc., M.Phil.)

## GENERAL INSTRUCTIONS :-

CODE:- AG-11

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A,B,C and D. Section - A comprises of 8 question of 1 mark each. Section - B comprises of 6 questions of 2 marks each. Section - C comprises of 10 questions of 3 marks each and Section - D comprises of 10 questions of 4 marks each.
3. Question numbers 1 to 8 in Sections - A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four mark each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 6 printed pages.

MATHEMATICS
CLASS X
Time : 3 to $31 / 4$ Hours
(SA-1)

SUMMATIVE ASSESSMENT -I (2013)

## SECTION A

Q. 1 If the HCF of 55 and 99 is expressible in the form $55 \mathrm{~m}-99$, then the value of $m$ is
(A) 4
(B) 2 (C) 1
(D)
3 Ans. B
Q. 2 The quadratic polynomial whose sum of zeroes is 3 and product of zeroes is -2 is
(A) $x^{2}+3 x-2(\mathrm{~B})$
(B) $x^{2}-2 x+3$
(C) $x^{2}-3 x+2$
(D) $x^{2}-3 x-2$ Ans. D
Q. 3 If in $\triangle A B C$ and $\triangle D E F, \frac{A B}{D E}=\frac{B C}{F D}$, then they will be similar triangles if
(A) $\angle B=\angle E$
(B) $\angle A=\angle D$
(C) $\angle B=\angle D(D)$
$\angle A=\angle F$ Ans. C
Q. 4 If $\cos \left(40^{\circ}+A\right)=\sin 30^{\circ}$, then value of A is
(A) $30^{\circ}(\mathrm{B})$
(B) $40^{\circ}$
(C) $60^{\circ}$ (D) $20^{\circ} \quad$ Ans. D
Q. 5

If $3 \cos \theta=2 \sin \theta$, then the value of $\frac{4 \sin \theta-3 \cos \theta}{2 \sin \theta+6 \cos \theta}$ is
(A) $\frac{1}{8}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$
Ans. B
Q. 6 Given that $\operatorname{LCM}(91,26)=182$, then $\operatorname{HCF}(91,26)$ is
(A) 13
(B) 26
(C) 7
(D) 9
Ans. A
Q. 7 One equation of a pair of dependent linear equations is $-5 x+7 y$ $=2$, the second equation can be
(A) $10 x+14 y+4=0$
(B) $-10 x-14 y+4=0$
(C) $-10 x+14 y+4=0$ (D) $10 x-14 y=-4$

Ans. D
Q. 8

If $\cos c \cot -\cot \theta=\frac{1}{3}$, then value of $(\operatorname{cosec} \theta+\cot \theta)$ is

$43,13,53,36$. If 13,23 is replace by 72,49 . what will be the new median . ans; median $=20$ and new median $=65 / 2$ ie $=32.5$
Q. 14 A vertical pole which is 2.25 m long cast a 6.75 m long shadow on the ground. At the same time, a vertical tower casts a 90 m long shadow on the ground. Find the height of the tower. ANS:

$\because$ At the same time the angle of elevation of the sum is same $\angle \mathrm{ACB}=\angle \mathrm{PRQ}$
$\angle \mathrm{ABC}=\angle \mathrm{PQR}=90^{\circ}$
By AA similarity $\triangle \mathrm{ABC} \square \triangle \mathrm{PQR}$

$$
\begin{aligned}
& \frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BC}}{\mathrm{QR}} \\
& \frac{2.25}{\mathrm{~h}}=\frac{6.75}{90} \\
& \mathrm{~h}=30 \mathrm{~m}
\end{aligned}
$$

## SECTION C

Q. 15 If $\operatorname{cosec}(A-B)=2, \cot (A+B)=\frac{1}{\sqrt{3}}, 0^{\circ}<(A+B) \leq 90^{\circ}$, then find A

$$
\operatorname{cosec}(A-B)=2 \text { and } \cot (A+B)=\frac{1}{\sqrt{3}}
$$

$\sin (A-B)=\frac{1}{2}$
$\mathrm{A}-\mathrm{B}=30^{\circ} \quad-\quad$ (1) $\quad \mathrm{A}+\mathrm{B}=60^{\circ} \quad$ - (2)
Solving (1) and (2) we get
$\mathrm{A}=45^{\circ}$ and $\mathrm{B}=15^{\circ}$
Q. 16 The sum of the numerator and denominator of a fraction is 12. If 1 is added to both the numerator and the denominator the fraction becomes $\frac{3}{4}$. Find the fraction.

Ans.

$$
\text { Let the fraction be } \frac{x}{y}
$$

According to the question
$x+y=12$
$\frac{x+1}{y+1}=\frac{3}{4} \Rightarrow 4 x-3 y=-1$
Solving and getting $x=5, y=7$
The fraction is $5 / 7$

## OR

A man travels 600 km partly by train and partly by car. It takes 8 hours and 40 minutes if he travels 320 km by train and the rest by car. It would take 30 minutes more if he travels 200 km by train and the rest by car. Find the speed of the train and the car


Speed of the train is $80 \mathrm{~km} / \mathrm{hr}$
Speed of the car is $60 \mathrm{~km} / \mathrm{hr}$
Q. 17 Prove that $\quad \frac{1+\cos A}{\sin A}+\frac{\sin A}{1+\cos A}=2 \operatorname{cosec} A \quad$ Ans.

LHS $=\frac{(1+\cos \mathrm{A})^{2}+\sin ^{2} \mathrm{~A}}{\sin \mathrm{~A}(1+\cos \mathrm{A})}$
$=\frac{1+2 \cos \mathrm{~A}+\cos ^{2} \mathrm{~A}+\sin ^{2} \mathrm{~A}}{\sin \mathrm{~A}(1+\cos \mathrm{A})}$
$=\frac{2(1+\cos \mathrm{A})}{\sin \mathrm{A}(1+\cos \mathrm{A})}$
$=\frac{2}{\sin \mathrm{~A}}=2 \operatorname{cosec} \mathrm{~A}=$ RHS
Q. 18 Find the zeroes of the quadratic polynomial $x^{2}+5 x+6$ and verify the relationship between the zeroes and the coefficients Ans. $f(x)=x^{2}+5 x+6=(x+3)(x+2)$
zeroes of polynomial arc -3 and -2
Sum of zeroes $=-5$ $\qquad$ (i)
$\frac{\text { coefficient of } x}{\text { coefficient of } x^{2}}=-\frac{5}{1}=-5$ $\qquad$ (ii)

$$
(\mathrm{i})=(\mathrm{ii})
$$

Product of zeroes $=(-3) \times(-2)=6$ $\qquad$ (iii)
$\frac{\text { Constant term }}{\text { Coefficient of } x^{2}}=\frac{6}{1}=6$ $\qquad$ (iv)
(iii) $=$ (iv)
Q. 19 On dividing the polynomial $4 x^{4}-5 x^{3}-39 x^{2}-46 x-2$ by the polynomial $g(x)$, the quotient and remainder were $x^{2}-3 x-5$ and $-5 x+8$ respectively. Find $g(x)$. Ans. $\mathrm{p}(x)=4 x^{4}-5 x^{3}-39 x^{2}-46 x-2$
$\mathrm{q}(x)=x^{2}-3 x-5$
$\mathrm{r}(x)=-5 x+8$
$\mathrm{g}(x)=\frac{\mathrm{p}(x)-\mathrm{r}(x)}{\mathrm{q}(x)}$
$=\frac{4 x^{4}-5 x^{3}-39 x^{2}-41 x-10}{x^{2}-3 x-5}$

$$
x^{2}-3 x-5 \underbrace{\frac{4 x^{4}-12 x^{3}-20 x^{2}}{7 x^{3}-19 x^{2}-41 x-10}}_{\frac{4 x^{2}+7 x+12}{4 x^{4}-5 x^{3}-39 x^{2}-41 x-10}} \begin{array}{r}
\frac{7 x^{3}-21 x^{2}-35 x}{2 x^{2}-6 x-10} \\
\frac{2 x^{2}-6 x-10}{0}
\end{array}
$$

$$
\mathrm{g}(x)=4 x^{2}+7 x+2
$$

Q. 20 Check whether $4^{n}$ can end with digit zero for any natural number n.ANS
If a number $4^{\text {11 }}$, for any natural number nends with digit 0 , then it is divisible by 5 .
The prime factorization of $4^{n}$ must contain the prime factor 5 .
This is not possible because prime factors of $4^{1 \pi}$ is 2 only and the uniqueness of
Fundamental theorem of arithmetic guarantees that there are no other prime in factorisation of $4^{17}$.
Hence $4^{1 \mathrm{c}}$ can never end with the digit zero for $n \in \mathrm{~N}$.
OR
Show that the square of any positive odd integer is of the form $8 \mathrm{~m}+1$, for some integer m . ANS

## Let ' $n$ ' be a positive odd Integer

Then $n=4 q+1$ or $4 q+3$

$$
\begin{aligned}
& n^{2}=(4 q+1)^{2} \text { or }(4 q+3)^{2} \\
& =8\left(2 q^{2}+q\right)+1 \text { or } 8\left(2 q^{2}+3 q+1\right)+1 \\
& =8 m+1 \text { where } m=\left(2 q^{2}+q\right) \text { or } m=2 q^{2}+3 q+1
\end{aligned}
$$

## Hence $n^{2}=8 m+1$ for some integer $m$.

The mean of the following data is 53 , find the missing frequencies.

| Classes | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |




In $\triangle \mathrm{MSR}$ and $\triangle \mathrm{MQP}$
$\angle 1=\angle 2$
$\angle 3=\angle 4$

$$
\begin{array}{ll}
\therefore & \Delta \mathrm{MSR} \sim \Delta \mathrm{MQP} \\
\Rightarrow & \frac{\mathrm{MS}}{\mathrm{MQ}}=\frac{\mathrm{MR}}{\mathrm{MP}} \\
\Rightarrow & \frac{\mathrm{MS}}{\mathrm{MR}}=\frac{\mathrm{MQ}}{\mathrm{MP}} \tag{1}
\end{array}
$$

Also, $\Delta \mathrm{PMS} \sim \Delta \mathrm{QMR}$

$$
\begin{equation*}
\Rightarrow \quad \frac{\mathrm{PM}}{\mathrm{QM}}=\frac{\mathrm{MS}}{\mathrm{MR}}=\frac{\mathrm{PS}}{\mathrm{QR}} \tag{2}
\end{equation*}
$$

From (1), (2) $\quad \frac{\mathrm{MQ}}{\mathrm{MP}}=\frac{\mathrm{PM}}{\mathrm{QM}}$

$$
\begin{array}{ll}
\Rightarrow & \mathrm{QM}^{2}=\mathrm{PM}^{2} \\
\Rightarrow & \mathrm{QM}=\mathrm{PM}
\end{array}
$$

(3)

From (1), (3) $\frac{\mathrm{MS}}{\mathrm{MR}}=1 \quad \Rightarrow \quad \mathrm{MS}=\mathrm{MR} \quad$ (4)

From (2), (3), (4) $\Rightarrow \quad \frac{\mathrm{PS}}{\mathrm{QR}}=1 \Rightarrow \mathrm{PS}=\mathrm{QR}$
Q. 23 In fig. PQR is a triangle in which $\mathrm{QM} \perp \mathrm{PR}$ and $P R^{2}-P Q^{2}=Q R^{2}$.

$$
\text { Prove that } Q M^{2}=P M \times M R
$$


.Ans.


$$
\begin{aligned}
& \mathbf{O R}{ }^{\mathrm{PR}^{2}=\mathrm{QR}^{2}+\mathrm{PQ}^{2} \Rightarrow \mathrm{PQR} \text { is right angled at } \mathrm{Q}} \\
& \angle 2+\angle 3=90^{\circ} ; \angle 1+\angle 2=90^{\circ} \Rightarrow \angle 1=\angle 3, \angle 2=\angle 4
\end{aligned}
$$

$$
\Delta \mathrm{PMQ} \sim \Delta \mathrm{QMR}(\mathrm{AA})
$$

$$
\frac{\mathrm{PM}}{\mathrm{QM}}=\frac{\mathrm{QM}}{\mathrm{MR}} \Rightarrow \mathrm{MQ}^{2}=\mathrm{PM} \times \mathrm{MR}
$$

In given figure ABCD is a parallelogram such that diagonals AC , $B D$ intersect at $O$. IF $P$ is mid-point of $C D$ and $C Q=\frac{1}{4} A C$. Prove

BC.


In a parallelogram, diagonals bisect each other

$$
\begin{equation*}
\mathrm{OC}=\frac{1}{2} \mathrm{AC} \tag{1}
\end{equation*}
$$

Now, in $\triangle B C D, P$ is mid-point of CD

$$
\begin{aligned}
\text { and } C Q=\frac{1}{4} A C & \Rightarrow C Q=\frac{1}{2}\left(\frac{1}{2} A C\right) \\
& \Rightarrow \quad C Q=\frac{1}{2} O C \\
& \Rightarrow \quad Q \text { is mid-point of } O C
\end{aligned}
$$

Now in $\triangle D O C, P$ is mid-point of $D C$ and $Q$ is mid-point of $O C$
$\Rightarrow \quad \frac{C P}{P D}=\frac{C Q}{Q O}$
$\Rightarrow \quad \mathrm{PQ} \| \mathrm{DO}$
[ By converse of BPT ]
$\Rightarrow \quad \mathrm{PR} \| \mathrm{DB}$
In $\triangle C D B, P R \| D B$
$\Rightarrow \quad \frac{C P}{P D}=\frac{C R}{R B}$
But $\quad \frac{C P}{P D}=1$

|  | $\begin{array}{ll} \Rightarrow & \frac{C R}{R B}=1 \\ \Rightarrow & C R=R B \end{array}$ <br> ANS <br> In $\triangle A C D$ and $\triangle A E C$ $\begin{array}{lll}  & \angle \mathrm{ACD}=\angle \mathrm{AEC}=90^{\circ} & \\ & \angle \mathrm{CAD}=\angle \mathrm{EAC} & \text { [ common ] } \\ \therefore \quad & \triangle \mathrm{ACD} \sim \triangle \mathrm{AEC} & \text { [AA similarity ] } \\ \therefore & \frac{\mathrm{AC}}{\mathrm{AE}}=\frac{\mathrm{AD}}{\mathrm{AC}} &  \tag{2}\\ \Rightarrow \quad & \mathrm{AC}^{2}=\mathrm{AE} \cdot \mathrm{AD} & \end{array}$ |
| :---: | :---: |
| Q. 24 | For which values of $a$ and $b$ does the following pair of linear equations have an infinite number of solutions? $(a-b) x+(a+b) y=3 a+b-2 \quad ; \quad 2 x+3 y=7$. <br> Ans. $a=5, b=1$ |
|  | SECTION D |
| Q. 25 | In Figure, BL and $C M$ are medians of $\triangle A B C$ right angled at $A$. <br> Ans. |


|  | In $\triangle$ BAL $\begin{align*} & \mathrm{BL}^{2}=\left(\frac{\mathrm{AC}}{2}\right)^{2}+\mathrm{AB}^{2} \\ & 4 \mathrm{BL}^{2}=\mathrm{AC}^{2}+4 \mathrm{AB}^{2} \tag{1} \end{align*}$ <br> In $\triangle C A M$ $\begin{aligned} & \mathrm{CM}^{2}=\mathrm{CA}^{2}+\frac{\mathrm{AB}^{2}}{4} \Rightarrow 4 \mathrm{CM}^{2}=4 \mathrm{CA}^{2}+\mathrm{AB}^{2} . \\ & (1)+(2) \\ & 4\left(\mathrm{BL}^{2}+\mathrm{CM}^{2}\right)=5\left[\mathrm{AC}^{2}+\mathrm{AB}^{2}\right] \\ & =5 \mathrm{BC}^{2}[\mathrm{By} \text { Pythagorous theorem }] \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. 26 | Solve graphically the pair of liner equations. <br> $x-y=-1$ and $2 x+y-10=0$. Also find the area of the region bounded by these lines and $x$-axis. Ans. Tabular column <br> Plotting the points and drawing <br> The correct graph <br> Solution $x=3 y=4$ and <br> Area $=12$ sq. units |  |  |  |  |  |
| Q. 27 | Find the median by drawing both types of ogives. |  |  |  |  |  |
|  | Classes | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |



## Correct Less than ogive

Correct More than ogive
Corrected Located median
Q. 28 Divide $2 x^{4}-9 x^{3}+5 x^{2}+3 x-8$ by $x^{2}-4 x+1$ and verify the division algorithm. Ans. $\left(2 x^{4}-9 x^{3}+5 x^{2}+3 x-8\right) \div\left(x^{2}-4 x+1\right)$ gives
$2 x^{2}-x-1$ as quotient and -7 as remainder
Verification, (Quotient $\times$ Divisor) + Remainder $=$ Divident
Q. 29 An army contingent of 616 members is to march behind and army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? ANS

| Given expression $=$ For each conversion $-1 / 2$ <br> $\sin 12^{\circ} \cos 15^{\circ} \sec 15^{\circ} \operatorname{cosec} 12^{\circ}$ $\frac{\sin ^{2}\left(40^{\circ}-\theta\right)+\cos ^{2}\left(40^{\circ}-\theta\right)}{\tan 15^{\circ} \tan 37^{\circ} \cot 37^{\circ} \cot 15^{\circ}}$ <br> For using the identity  |  |
| :--- | :--- |
| $=\frac{-1}{1}+\frac{1}{1}=0$ | For final answer |

Q. 31 In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle. Prove it . Given : A triangle ABC such that $A C^{2}=A B^{2}+B C^{2}$


Construction : Construct a triangle DEF such that $\mathrm{DE}=\mathrm{AB}, \mathrm{EF}$ $=\mathrm{BC}$ and $\angle \mathrm{E}=90^{\circ}$
Proof : In order to prove that $\angle \mathrm{B}=90$. 0 , it is sufficient to show $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$. For this we proceed as follows Since $\triangle$ DEF is a right - angled triangle with right angle at E. Therefore, by Pythagoras theorem, we have

$$
\begin{aligned}
& \mathrm{DF}^{2}=\mathrm{DE}^{2}+\mathrm{EF}^{2} \\
& \Rightarrow \mathrm{DF}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}[\therefore \mathrm{DE}=\mathrm{AB} \text { and } \mathrm{EF}=\mathrm{BC}(\mathrm{By}
\end{aligned}
$$



## OR

If two zeroes of the polynomial $p(x)=$

|  | $x^{4}-6 x^{3}-26 x^{2}+138 x-35$ are $2 \pm \sqrt{3}$, find the other zeroes. ANS :$\text { Sum }=4 \text { Product }=1$$\begin{gathered} \left(x^{4}-6 x^{3}-26 x^{2}+138 x-35\right) \div\left(x^{2}-4 x+1\right) \\ =x^{2}-2 x-35 \end{gathered}$$\Rightarrow \text { Polynomial of is } x-+4 x+1 \quad \Rightarrow \text { zeroes are } 7,-5$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. 33 | Find the mean of the following data using step up deviation method . |  |  |  |  |  |  |  |
|  | C.I 2 | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60 |
|  | f | 14 | 22 | 16 | 6 | 5 | 3 | 4 |
|  | class |  |  | $\mathrm{d}_{i}$ | $x_{i}-\mathrm{a}$ | $u_{i}=\frac{x_{i}}{1}$ |  | $f_{i} u_{i}$ |
|  | 25-30 |  |  | 14 | -15 | -3 |  | -42 |
|  | 30-35 |  |  | 22 | - 10 | -2 |  | -44 |
|  | 35-40 |  |  | 16 | - 5 | -1 |  | - 16 |
|  | 40-45 |  | . $5=\mathrm{a}$ | 6 | o | o |  | o |
|  | 45-50 |  | . 5 | 5 | 5 | 1 |  | 5 |
|  | 50-55 |  |  | 3 | 10 | 2 |  | 6 |
|  | 55-60 |  |  | 4 | 15 | 3 |  | 12 |
|  | $\begin{array}{r} \mathrm{a}=42.5 \\ \bar{x}=\mathrm{a} \end{array} \mathrm{+}, \mathrm{~h},$ | $\begin{aligned} & .5, \mathrm{~h}=5 \\ & \mathrm{~h} \frac{\Sigma f_{i} u_{i}}{\Sigma f_{i}} \\ & =42.5+5 \\ & =36.86 \end{aligned}$ | $\times \frac{(-79)}{70}$ | $=70$ |  |  |  | $u_{i}=-79$ |
| Q. 34 | Prove | that: |  |  |  |  |  |  |


|  | $\begin{aligned} &\left(\begin{array}{rl} (\sin \theta+\sec \theta)^{2} & +(\cos \theta+\cos e c \theta)^{2}=(1+\sec \theta \cos e c \theta \end{array}\right) \cdot \underline{\mathbf{A}} \\ & \text { Consider }(\sin \theta+\sec \theta)^{2}+(\cos \theta+\operatorname{cosec} \theta)^{2} \\ &=\sin ^{2} \theta+\sec ^{2} \theta+2 \tan \theta+\cos ^{2} \theta+\operatorname{cosec}^{2} \theta+2 \cot \theta \\ &=1+\frac{\cos ^{2} \theta+\sin ^{2} \theta}{\sin ^{2} \theta \cos ^{2} \theta}+2\left[\frac{\sin \theta}{\cos \theta}+\frac{\cos \theta}{\sin \theta}\right] \\ &=1+\sec ^{2} \theta \operatorname{cosec} 2 \theta+2 \sec \theta \operatorname{cosec} \theta \\ &=(1+\sec \theta \operatorname{cosec} \theta)^{2} \end{aligned}$ |
| :---: | :---: |
|  | ***************** |
|  | HAPPINESS IS NOTHING MORE THAN GOOD HEALTH AND A BAD MEMORY. |

